Quick Review Sheet Math 1324

Symmetry

Algebraic Test of Symmetry

***x-axis**:* If replacing y with $-y$ produces an equivalent equation, then the graph is *symmetric with respect to the x-axis.*

***y-axis**:* If replacing x with $-x$ produces an equivalent equation, then the graph is *symmetric with respect to the y-axis.*

***Origin**:* If replacing x with $-x$ and y with $-y$ produces an equivalent equation, then the graph is *symmetric with respect to the origin.*

Even and Odd Functions

If the graph of a function f is symmetric with respect to the y-axis, we say that it is an **even function**. That is, for each x in the domain of f, $f(x) = f(-x)$.

If the graph of a function f is symmetric with respect to the origin, we say that it is an **odd function.** That is, for each x in the domain of f, $f(-x) = -f(x)$.

Transformations

Vertical Translation: $y = f(x) \pm b$

For b > 0,

the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted *up* b units;
the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted *down* b units.

Horizontal Translation: $y = f(x \pm d)$

For $d > 0$,

the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted *right* d units;

the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted *left* d units.

Reflections

Across the x-axis: The graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the x-axis.

Across the y-axis: The graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ across the y-axis.

Vertical Stretching and Shrinking: $y = a\,f(x)$

The graph of $y = a\,f(x)$ can be obtained from the graph of $y = f(x)$ by

stretching vertically for $|a| > 1$, or
shrinking vertically for $0 < |a| < 1$

For $a < 0$, the graph is also reflected across the x-axis.

Horizontal Stretching or Shrinking: $y = f(cx)$

The graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by

shrinking horizontally for $|c| > 1$, or
stretching horizontally for $0 < |c| < 1$.

For c < 0, the graph is also reflected across the y-axis.

Quadratic Formula

The solutions of $ax^2 + bx + c = 0,\ a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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The Vertex of a Parabola

The **vertex** of the graph of $f(x) = ax^2 + bx + c$ is

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

We calculate the x-coordinate (for $-\frac{b}{2a}$); we substitute to find the y-coordinate (for $f\left(-\frac{b}{2a}\right)$).

The Algebra of Functions

The Sums, Differences, Products, and Quotients of Functions

If f and g are functions and x is the domain of each function, then

$(f + g)(x) = f(x) + g(x)$
$(f - g)(x) = f(x) - g(x)$
$(fg)(x) = f(x) \cdot g(x)$
$(f/g)(x) = f(x)/g(x)$, provided $g(x) \neq 0$

Composition of Functions

The **composition function** $f \circ g$, the **composition** of f and g, is defined as

$$(f \circ g)(x) = f(g(x)),$$

where x is in the domain of g and $g(x)$ is in the domain of f.

One-to-One Functions

A function f is **one-to-one** if different inputs have different outputs—that is,

if $a \neq b$, then $f(a) \neq f(b)$

Or a function f is **one-to-one** if when the outputs are the same, the inputs are the same—that is,

if $f(a) = f(b)$, then $a = b$

Horizontal-Line Test

If it is possible for a horizontal line to intersect the graph of a function more than once, then the function is *not* one-to-one and its inverse is *not* a function.

Obtaining a Formula for an Inverse

If a function f is one-to-one, a formula for its inverse can generally be found as follows:

1. Replace $f(x)$ with y.
2. Intercharge x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Exponential and Logarithmic Functions

The function $f(x) = a^x$, where x is a real number, $a > 0$ and $a \neq 1$, is called the **exponential function,** base a.

We define $y = \log_a x$ as that number y such that $x = a^y$, where $x > 0$ and a is a positive constant other than 1.

Summary of the Properties of Logarithms

Product Rule: $\log_a MN = \log_a M + \log_a N$
Power Rule: $\log_a M^p = p \cdot \log_a M$
Quotient Rule: $\log_a \frac{M}{N} = \log_a M - \log_a N$
Change-of-Base Formula: $\log_b M = \frac{\log M}{\log b}$

Other Properties:

$\log_a a = 1$ $\quad$ $\log_a 1 = 0$

$\log_a a^x = x$ $\quad$ $a^{\log_a x} = x$

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Solving Exponential and Logarithmic Equations

Base-Exponent Property

For any $a > 0$, $a \neq 1$,

$$a^x = a^y \leftrightarrow x = y$$

Property of Logarithmic Equality

For any $M > 0, N > 0, a > 0$, and $a \neq 1$,

$$\log_a M = \log_a N \leftrightarrow M = N$$

A Logarithm is an Exponent

$$\log_a x = y \leftrightarrow x = a^y$$

Mathematics of Finance

Simple Interest: $I = Prt$, **where**

I = interest P = principal
r = annual simple interest rate (written as a decimal)
t = time in years

Amount: Simple Interest: $A = P + Prt$, **where**

A = amount, or future value t = time in years
P = principal, or present value
r = annual simple interest rate (written as a decimal)

Compound Interest: $A = P(1+i)^n$, **where** $i = r/m$ **and**

A = amount (future value) at the end of n periods
P = principal (present value)
r = annual rate
m = number of compounding periods per year
i = rate per compounding period
n = total number of compounding

Continuous Compound Interest Formula

$A = Pe^{rt}$ where

A = amount in the account after t years
P = Principal, or present value
r = interest rate (written as a decimal)
t = time in years

Annual Percentage Yield

If a principal is invested at the annual rate r compounded m times a year, then the annual percentage yield is

$$\text{APY} = \left(1 + \frac{r}{m}\right)^m - 1$$

If a principal is invested at the annual rate r compounded continuously, then the annual percentage yield is

$$\text{APY} = e^r - 1$$

Future Value of an Ordinary Annuity

$$FV = PMT\frac{(1+i)^n - 1}{i}$$ where

FV = future value (amount)
PMT = periodic payment
i = rate per period
n = number of payments (periods)

Present Value of an Ordinary Annuity

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$ where

PV = present value of all payments
PMT = periodic payment
i = rate per period
n = number of periods

Strategy for Solving Mathematics of Finance Problems

Step 1. Determine whether the problem involves a single payment or a sequence of equal periodic payments. Simple and compound interest problems involve a single present value and a single future value. Ordinary annuities may be concerned with a present value or a future value but always involve a sequence of equal periodic payments.

Step 2. If a single payment is involved, determine whether simple or compound interest is used. Simple interest is usually used for durations of a year or less and compound interest for longer periods.

Step 3. If a sequence of periodic payments is involved, determine whether the payments are being made into an account that is increasing in value—a future value problem—or the payments are being made out of an account that is decreasing in value—a present value problem. Remember that amortization problems always involve the present value of an ordinary annuity.