FUNDAMENTAL ARITHMETIC

Prime Numbers

Prime numbers are any whole numbers greater than 1 that can only be divided by 1 and itself. Below is the list of all prime numbers between 1 and 100:

2	3	5	7	11	13	17	19	23	29	31	37	41
43	47	53	59	61	67	71	73	79	83	89	97	

Prime Factorization

Prime factorization is the process of changing non-prime (composite) numbers into a product of prime numbers. Below is the prime factorization of all numbers between 1 and 20.

1 is unique	$6 = 2 \cdot 3$	11 is prime	$16 = 2 \cdot 2 \cdot 2 \cdot 2$
2 is prime	7 is prime	$12 = 2 \cdot 2 \cdot 3$	17 is prime
3 is prime	$8 = 2 \cdot 2 \cdot 2$	13 is prime	$18 = 2 \cdot 3 \cdot 3$
$4 = 2 \cdot 2$	$9 = 3 \cdot 3$	$14 = 2 \cdot 7$	19 is prime
5 is prime	$10 = 2 \cdot 5$	$15 = 3 \cdot 5$	$20 = 2 \cdot 2 \cdot 5$

Divisibility

By using these divisibility rules, you can easily test if one number can be evenly divided by another.

A Number is Divisible By:	If:	Examples
2	The last digit is divisible by two	5836 ~~583~~ **6** **6** $\div 2 = 3$, Yes
3	The sum of the digits is divisible by three	1725 $1 + 7 + 2 + 5 =$ **15** **15** $\div 3 = 5$, Yes
5	The last digit ends in a five or zero	885 ~~88~~ **5** The last digit ends in a five, Yes
10	The last digit ends in a zero	7390 ~~739~~ **0** The last digit ends in a zero, Yes

Exponents

An exponent of a number says how many times to multiply the base number to itself.

$$\underset{\uparrow \atop base}{5}{}^{3 \leftarrow exponent} \qquad 2^6 \qquad 7^5$$

$$\underbrace{5 \times 5 \times 5}_{three\ 5s\ multiplied\ together} \qquad \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}_{six\ 2s\ multiplied\ together} \qquad \underbrace{7 \times 7 \times 7 \times 7 \times 7}_{five\ 7s\ multiplied\ together}$$

$$125 \qquad 64 \qquad 16807$$

Least Common Multiple

The Least Common Multiple (LCM) is the smallest positive number that is a multiple of two or more numbers. To find the LCM of some numbers:

1) express each of the numbers as a product of its prime factors,	$50 = 2 \cdot 5 \cdot 5$	$42 = 2 \cdot 3 \cdot 7$	$36 = 2 \cdot 2 \cdot 3 \cdot 3$
2) re-write each of the factors using exponents	$50 = 2^1 \cdot 5^2$	$42 = 2^1 \cdot 3^1 \cdot 7^1$	$36 = 2^2 \cdot 3^2$
3) for each base, select/circle the largest power among all of the factors	$50 = 2^1 \cdot \mathbf{5^2}$	$42 = 2^1 \cdot 3^1 \cdot \mathbf{7^1}$	$36 = \mathbf{2^2} \cdot \mathbf{3^2}$
4) multiply the selected powers together	$2^2 \cdot 3^2 \cdot 5^2 \cdot 7^1 = \mathbf{6300}$		

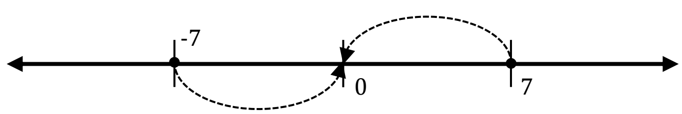
Greatest Common Factor

The Greatest Common Factor (GCF) is the largest number that divides evenly into two or more numbers. To find the GCF of some numbers:

1) express each of the numbers as a product of its prime factors,	$50 = 2 \cdot 5 \cdot 5$	$42 = 2 \cdot 3 \cdot 7$	$36 = 2 \cdot 2 \cdot 3 \cdot 3$
2) re-write each of the factors using powers	$50 = 2^1 \cdot 5^2$	$42 = 2^1 \cdot 3^1 \cdot 7^1$	$36 = 2^2 \cdot 3^2$
3) for each base common to all factorizations, select/circle the smallest power among all of the factors	$50 = \mathbf{2^1} \cdot 5^2$	$42 = \mathbf{2^1} \cdot 3^1 \cdot 7^1$ $\mathbf{2^1}$ 2^1 2^2	$36 = \mathbf{2^2} \cdot 3^2$
4) multiply the selected powers together	$2^1 = \mathbf{2}$		

Introduction to Integers

Integers are an extension of whole numbers. They include whole numbers ($\{0,1,2,3,\ldots\}$) and their opposites ($\{-1,-2,-3,\ldots\}$). Two integers are opposites if they are each the same distance away from zero, but on opposite sides of zero.



7 and (-7) are opposites.

Absolute Value

The absolute value of a number may be thought of as its distance from zero. The absolute value of a number will be *either positive or zero* (non-negative).

$\lvert 17 \rvert = 17$	$\lvert -21 \rvert = 21$	$\lvert 0 \rvert = 0$	$-\lvert -3 \rvert = -3$
$\lvert 9 + 2 \rvert = \lvert 11 \rvert = 11$	$\lvert 23 - 23 \rvert = \lvert 0 \rvert = 0$	$-\lvert 14 - 19 \rvert = -\lvert -5 \rvert = -5$	$\lvert 6 - 12 \rvert = \lvert -6 \rvert = 6$

Adding and Subtracting Integers

The difference of two integers should be rewritten as the sum of the first integer and the opposite of the second. Then, follow the appropriate rule below for adding integers.

The sum of two integers with the <u>same sign</u> is their sum with their shared sign.			The sum of two integers with <u>different signs</u> is their difference, with the sign of the integer with the largest absolute value.		
$-13 - 12$		$15 - (-7)$		$-12 - (-9)$	
$-13 + (-12)$	$-14 + (-3)$	$15 + 7$	$-3 + 13$	$-12 + 9$	$-11 + 2$
			$\lvert -3 \rvert < \lvert 13 \rvert$	$\lvert -12 \rvert > \lvert 9 \rvert$	$\lvert -11 \rvert > \lvert 2 \rvert$
$-(13 + 12)$	$-(14 + 3)$	$+(15 + 7)$	$+(13 - 3)$	$-(12 - 9)$	$-(11 - 2)$
-25	-17	22	10	-3	-9

Multiplying and Dividing Integers

The product or quotient of two integers with the same sign is positive.			The product or quotient of two integers with different signs is negative.		
$(-17)(-7)$	$(14)(6)$	$-65 \div (-13)$	$14 \div (-7)$	$(-4)(12)$	$-33 \div 3$
119	84	5	-2	-48	-11

Order of Operations

The order of operations is a rule that lets us know which operations should be performed first. This ordering is as follows:

- Parentheses (Brackets and Braces) $\{\ [\ (\quad)\ ]\ \}$
- Exponents $4^2\ \sqrt[3]{27}\ \sqrt{64}$
- Multiplication and Division (from left to right) $\times\ or\ \div$
- Addition and Subtraction (from left to right) $+\ or\ -$

A group of numbers and operations inside of a parentheses will also follow the order of operations.

Introduction to Fractions

A fraction is another way of representing division. The bottom value of a fraction (the denominator) describes the number of equal parts that divide the whole. The top value (the numerator) describes how many of those parts there are. A proper fraction represents part of a whole and it has a value that is less than one and greater than zero.

$$\begin{array}{c} numerator \\ \dfrac{2}{5} \\ denominator \end{array}$$

Equivalent Fractions

Equivalent fractions have the same overall value, but the values of the numerators and denominators are different for each fraction. To make an equivalent fraction, we can either multiply or divide both the numerator and denominator of the fraction by the same number.

$$\frac{2}{5} = \frac{2 \times 1}{5 \times 1} = \frac{2}{5} \qquad \frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15} \qquad \frac{18}{36} = \frac{18 \div 1}{36 \div 1} = \frac{18}{36} \qquad \frac{18}{36} = \frac{18 \div 3}{36 \div 3} = \frac{6}{12}$$

$$\frac{2}{5} = \frac{2 \times 5}{5 \times 5} = \frac{10}{25} \qquad \frac{2}{5} = \frac{2 \times 10}{5 \times 10} = \frac{20}{50} \qquad \frac{18}{36} = \frac{18 \div 6}{36 \div 6} = \frac{3}{6} \qquad \frac{18}{36} = \frac{18 \div 18}{36 \div 18} = \frac{1}{2}$$

Simplifying Fractions

In order to simplify a fraction, perform the prime factorization on the numerator and denominator of the fraction. Then, each common factor in both the numerator and denominator can be divided away.

$$\frac{60}{42}$$

$$\frac{\mathbf{2} \times 2 \times \mathbf{3} \times 5}{\mathbf{2} \times \mathbf{3} \times 7}$$

$$\frac{\mathbf{2}}{\mathbf{2}} \times \frac{\mathbf{3}}{\mathbf{3}} \times \frac{2 \times 5}{7}$$

$$1 \times 1 \times \frac{2 \times 5}{7}$$

$$\frac{2 \times 5}{7}$$

$$\frac{10}{7}$$

$$\frac{135}{162}$$

$$\frac{\mathbf{3} \times \mathbf{3} \times \mathbf{3} \times 5}{2 \times \mathbf{3} \times \mathbf{3} \times \mathbf{3} \times 3}$$

$$\frac{\mathbf{3}}{\mathbf{3}} \times \frac{\mathbf{3}}{\mathbf{3}} \times \frac{\mathbf{3}}{\mathbf{3}} \times \frac{5}{2 \times 3}$$

$$1 \times 1 \times 1 \times \frac{5}{2 \times 3}$$

$$\frac{5}{2 \times 3}$$

$$\frac{5}{6}$$

Least Common Denominator

The Least Common Denominator (LCD) is the smallest positive number that is a multiple of two or more denominators. To find the LCD of some numbers:

1) express each of the denominators as a product of its prime factors,	$28 = 2 \cdot 2 \cdot 7$ $42 = 2 \cdot 3 \cdot 7$ $12 = 2 \cdot 2 \cdot 3$
2) re-write each of the factors using powers	$28 = 2^2 \cdot 7^1$ $42 = 2^1 \cdot 3^1 \cdot 7^1$ $12 = 2^2 \cdot 3^1$
3) for each base, select/circle the largest power among all of the factors	$28 = \mathbf{2^2} \cdot \mathbf{7^1}$ $42 = 2^1 \cdot \mathbf{3^1} \cdot 7^1$ $12 = 2^2 \cdot 3^1$
4) multiply the selected powers together	$2^2 \cdot 3^1 \cdot 7^1 = \mathbf{84}$

Multiplying and Dividing Fractions

Given two fractions, multiplication can be written in three different ways.

$$\left(\frac{3}{10}\right)\left(\frac{1}{4}\right) \qquad \frac{3}{10} \times \frac{1}{4} \qquad \frac{3}{10} \cdot \frac{1}{4}$$

$$\frac{3}{40} \qquad \frac{3}{40} \qquad \frac{3}{40}$$

To multiply fractions, multiply the numerators together and then multiply the denominators together. Then, simplify if possible.

$$\left(\frac{3}{5}\right)\left(\frac{3}{7}\right) \quad \frac{3 \times 3}{5 \times 7} \quad \frac{9}{35}$$

$$\left(\frac{2}{3}\right)\left(\frac{1}{6}\right) \quad \frac{2 \times 1}{3 \times 6} \quad \frac{2}{18} \quad \frac{1}{9}$$

$$\left(\frac{3}{4}\right)\left(\frac{4}{10}\right) \quad \frac{3 \times 4}{4 \times 10} \quad \frac{12}{40} \quad \frac{3}{10}$$

$$\left(\frac{7}{9}\right)\left(\frac{3}{7}\right) \quad \frac{7 \times 3}{9 \times 7} \quad \frac{21}{63} \quad \frac{1}{3}$$

To divide fractions, *swap the values of numerator and denominator* (take the reciprocal) of the second fraction and then multiply the fractions.

$$\frac{5}{6} \div \frac{6}{7} \quad \left(\frac{5}{6}\right)\left(\frac{7}{6}\right) \quad \frac{5 \times 7}{6 \times 6} \quad \frac{35}{36}$$

$$\frac{1}{2} \div \frac{7}{8} \quad \left(\frac{1}{2}\right)\left(\frac{8}{7}\right) \quad \frac{1 \times 8}{2 \times 7} \quad \frac{8}{14} \quad \frac{4}{7}$$

$$\frac{3}{4} \div \frac{7}{10} \quad \left(\frac{3}{4}\right)\left(\frac{10}{7}\right) \quad \frac{3 \times 10}{4 \times 7} \quad \frac{30}{28} \quad \frac{15}{14}$$

Comparing Fractions

To compare fractions, find the least common denominator (LCD) of the fractions and rewrite each fraction using that LCD. Then, the numerators can be compared.

$$\frac{3}{4},\frac{7}{12},\frac{10}{13}$$

$4 = \mathbf{2^2}$ $\quad 12 = 2^2 \times \mathbf{3^1}$ $\quad 13 = \mathbf{13^1}$

$$\mathbf{2^2 \times 3^1 \times 13^1 = 156}$$

LCD(4,12,13) is **156**

$$\frac{3}{4}\left(\frac{\mathbf{3}}{\mathbf{3}}\right)\left(\frac{\mathbf{13}}{\mathbf{13}}\right), \quad \frac{7}{12}\left(\frac{\mathbf{13}}{\mathbf{13}}\right), \quad \frac{10}{13}\left(\frac{\mathbf{12}}{\mathbf{12}}\right)$$

$$\frac{117}{156}, \quad \frac{91}{156}, \quad \frac{120}{156}$$

$$\frac{1}{3},\frac{3}{8},\frac{4}{12}$$

$3 = \mathbf{3^1}$ $\quad 8 = \mathbf{2^3}$ $\quad 12 = 2^2 \times 3^1$

$$\mathbf{2^3 \times 3^1 = 24}$$

LCD(3,8,12) is **24**

$$\frac{1}{3}\left(\frac{\mathbf{8}}{\mathbf{8}}\right), \quad \frac{3}{8}\left(\frac{\mathbf{3}}{\mathbf{3}}\right), \quad \frac{4}{12}\left(\frac{\mathbf{2}}{\mathbf{2}}\right)$$

$$\frac{8}{24}, \quad \frac{9}{24}, \quad \frac{8}{24}$$

Adding and Subtracting Fractions

To add or subtract fractions with the same denominators, add or subtract the numerators and leave the denominators alone.

$\frac{3}{5}+\frac{2}{5}$	$\frac{8}{11}-\frac{2}{11}$	$\frac{3}{5}-\frac{3}{5}$	$\frac{11}{15}+\frac{7}{15}$	$\frac{9}{4}-\frac{5}{4}$	$\frac{5}{13}+\frac{2}{13}$	$\frac{11}{12}-\frac{8}{12}$	$\frac{3}{7}+\frac{2}{7}$
$\frac{5}{5}$	$\frac{6}{11}$	$\frac{0}{5}$	$\frac{18}{15}$	$\frac{4}{4}$	$\frac{7}{13}$	$\frac{3}{12}$	$\frac{5}{7}$
1		0	$\frac{6}{5}$	1		$\frac{1}{4}$	

To add or subtract fractions with different denominators, we will find a common denominator, and then change one or both of the fractions to have that common denominator.

$$\frac{3}{4}+\frac{7}{12}$$

$4 = \mathbf{2^2};\ 12 = 2^2 \times \mathbf{3^1};$

$$\mathbf{2^2 \times 3^1 = 12}$$

LCD(4,12) is **12**

$$\frac{3}{4}\left(\frac{\mathbf{3}}{\mathbf{3}}\right)+\frac{7}{12}\left(\frac{\mathbf{1}}{\mathbf{1}}\right)$$

$$\frac{9}{12}+\frac{7}{12}$$

$$\frac{16}{12}=\frac{4}{3}$$

$$\frac{1}{6}+\frac{3}{8}$$

$6 = 2^1 \times \mathbf{3^1};\ 8 = \mathbf{2^3};$

$$\mathbf{3^1 \times 2^3 = 24}$$

LCD(6,8) is **24**

$$\frac{1}{6}\left(\frac{\mathbf{4}}{\mathbf{4}}\right)+\frac{3}{8}\left(\frac{\mathbf{3}}{\mathbf{3}}\right)$$

$$\frac{4}{24}+\frac{9}{24}$$

$$\frac{13}{24}$$

$$\frac{4}{3}-\frac{4}{5}$$

$3 = \mathbf{3^1};\ 5 = \mathbf{5^1};$

$$\mathbf{5^1 \times 3^1 = 15}$$

LCD(3,5) is **15**

$$\frac{4}{3}\left(\frac{\mathbf{5}}{\mathbf{5}}\right)-\frac{4}{5}\left(\frac{\mathbf{3}}{\mathbf{3}}\right)$$

$$\frac{20}{15}-\frac{12}{15}$$

$$\frac{8}{15}$$

billions		hundred millions	ten millions	millions		hundred thousands	ten thousands	thousands		hundreds	tens	ones		tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths	ten millionths	hundred millionths	billionths	ten billionths	hundred billionths	trillionths	ten trillionths	hundred trillionths
1	,	5	7	8	,	4	3	8	,	0	5	0	.	2	5	1	6	9	4	0	7	9	9	6	0	8	7
10^9		10^8	10^7	10^6		10^5	10^4	10^3		10^2	10^1	10^0		$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	$\frac{1}{10^5}$	$\frac{1}{10^6}$	$\frac{1}{10^7}$	$\frac{1}{10^8}$	$\frac{1}{10^9}$	$\frac{1}{10^{10}}$	$\frac{1}{10^{11}}$	$\frac{1}{10^{12}}$	$\frac{1}{10^{13}}$	$\frac{1}{10^{14}}$

A decimal is a number with an integer part that is placed to the left of a decimal point, and a fractional part that is placed to the right. For example, in the number above, the hundredths place has the value of $5 \times \frac{1}{10^2}$ and the ten billionths place has the value of $9 \times \frac{1}{10^{10}}$.

Adding and Subtracting Decimals

To add decimals, begin by lining up the decimal points of the two numbers so that the common place values can be added together. If necessary, insert zeros to have the same number of digits for both numbers. Subtraction of decimals works the same way as addition does.

$$\begin{array}{rr} & 261.460 \\ & 3.900 \\ + & .324 \\ \hline & 265.684 \end{array} \qquad \begin{array}{rr} & 91.37 \\ + & 6.79 \\ \hline & 84.58 \end{array} \qquad \begin{array}{rr} & 431.135 \\ & 13.700 \\ + & 6.280 \\ \hline & 451.115 \end{array}$$

Multiplying Decimals

To multiply decimals, multiply the numbers while ignoring the decimal points. Then put the decimal point in the product so that the answer will have as many decimal places as the two original numbers combined.

$$\begin{array}{rrl} & 2.3 & \text{one decimal place} \\ \times & .14 & \text{two decimal places} \\ \hline & 92 & \\ & 230 & \\ \hline & .322 & \text{three decimal places} \end{array} \qquad \begin{array}{rr} & 61.52 \\ \times & .41 \\ \hline & 6152 \\ & 246080 \\ \hline & 25.2232 \end{array}$$

Dividing Decimals

To divide decimals, begin by moving the decimal point of both numbers to the right until the number you are dividing by (divisor/denominator) becomes a whole number. Then divide normally and place the decimal point in the same position as the number you are dividing (dividend/numerator).

$$.25 \overline{)\,1.5625} \qquad \begin{array}{r} 6.25 \\ 25\,\overline{)\,156.25} \\ -\,150 \\ \hline 62 \\ -\,50 \\ \hline 125 \\ -\,125 \\ \hline 0 \end{array} \qquad 1.4 \overline{)\,.322} \qquad \begin{array}{r} .23 \\ 14\,\overline{)\,3.22} \\ -\,28 \\ \hline 42 \\ -\,42 \\ \hline 0 \end{array}$$

Both move over two decimal places

Introduction to Percents

The word percent means “parts per hundred”. Using the percent symbol (%) is a convenient way to write “out of one hundred”. For example, instead of saying "*39 out of every 100* college students prefer reading books versus writing papers", we can say "*39%* of college students prefer reading print books versus e-books".

Converting Fractions, Decimals, and Percents

To change a decimal to a fraction : $.\#\# \to \frac{\#}{\#}$

Write the decimal on the top of the fraction and ‘1’ on the bottom of the fraction. Count the number of digits after the decimal place and multiply the top and bottom by 10 for every digit, if necessary. Then simplify the fraction, if possible.

$.278$	$.78$	1.14
$\frac{.278}{1}$	$\frac{.78}{1}$	$\frac{1.14}{1}$
$\frac{.278(10)(10)(10)}{(10)(10)(10)}$	$\frac{.78(10)(10)}{(10)(10)}$	$\frac{1.14(10)(10)}{(10)(10)}$
$\frac{278}{1000}$	$\frac{78}{100}$	$\frac{114}{100}$
$\frac{139}{500}$	$\frac{39}{50}$	$\frac{57}{50}$

To change a percent to a fraction : $\#\#\ \% \to \frac{\#}{\#}$

Write the percent on the top of the fraction and ‘100’ on the bottom of the fraction then, remove the percent sign. Count the number of digits after the decimal place and multiply the top and bottom by 10 for every digit, if necessary. Then simplify the fraction, if possible.

$50\ \%$	$79.2\ \%$	$173\ \%$
$\frac{50}{100}$	$\frac{79.2}{100}$	
	$\frac{79.2(10)}{100(10)}$	
	$\frac{792}{1000}$	
$\frac{1}{2}$	$\frac{99}{125}$	$\frac{173}{100}$

To change a percent to a decimal : $\#\#\ \% \to .\#\#$

Divide the percent by 100 and remove the percent sign.

$46.1\ \%$	$190\ \%$	$94\ \%$
$46.1 \div 100$	$190 \div 100$	$94 \div 100$
$.461$	1.90	$.94$

To change a decimal to a percent : $.\#\# \to \#\#\ \%$

Multiply the decimal by 100 and add the percent sign.

0.636	$.3$	1.28
0.636×100	$.3 \times 100$	1.28×100
$63.6\ \%$	$30\ \%$	$128\ \%$

To change a fraction to a percent : $\frac{\#}{\#} \to \#\#\ \%$

Divide the numerator by the denominator, multiply by 100, and add a percent sign.

$\frac{8}{25}$	$\frac{6}{10}$	$\frac{15}{16}$
$8 \div 25$	$6 \div 10$	$15 \div 16$
$.32 \times 100$	$.6 \times 100$	$.9375 \times 100$
$32\ \%$	$60\ \%$	$93.75\ \%$

To change a fraction to a decimal : $\frac{\#}{\#} \to .\#\#$

Divide the numerator by the denominator.

$\frac{16}{10}$	$\frac{85}{50}$	$\frac{39}{20}$
$16 \div 10$	$85 \div 50$	$39 \div 20$
1.6	1.7	1.95